Minimum Cost Routing: Robustness through Randomization

Vladimir Marbukh[1]
National Institute of Standards and Technology
100 Bureau Drive, Stop 8920
Gaithersburg, MD 20899-8920
e-mail: marbukh@nist.gov

Abstract — **A minimum cost routing admits an arriving request on the minimum cost route if this minimum cost does not exceed the cost of the request, and rejects the request otherwise. Minimum cost routing strategies naturally arise as a result of optimization of the network performance or incorporating Quality of Service (QoS) requirements into the admission and routing processes. In the former case the implied cost of the resources represents the expected future revenue losses due to insufficient resources for servicing future requests. In the latter case the cost of a route represents the QoS provided to the request. In both cases due to the aggregation, delays in disseminating signaling information, statistical inferences, non-steady or adversarial operational environment the cost of the resources may not be known exactly. Usually this uncertainty is modeled by assuming that the resource costs are random variables with some fixed probability distributions. We propose to explore a different approach intended to guard against the worst case scenario with respect to the resource costs. We solve the corresponding game in a symmetric case.**

I. Instability of a Minimum Cost Routing

A minimum cost routing scheme makes the admission and routing decisions $r = r_{opt}$ for an arriving request by solving the following optimization problem:

$$r_{opt} = \arg \max_{r \in \{\varnothing, R\}} u(c, r \mid w) \tag{1}$$

where the set of feasible routes is $R = \{r_1, .., r_K\}$, selection $r = \varnothing$ means that the request is rejected, the vector of route costs is $c = (c_r : r \in R)$, utility of selection of a route $r \in \{\varnothing, R\}$ is

$$u(c, r \mid w) = \begin{cases} w - c_r & if \quad r \neq \varnothing \\ 0 & if \quad r = \varnothing \end{cases} \tag{2}$$

and the cost of the arriving request is w. The cost of a route c_r is usually additive: $c_r = \sum_{l \in r} c_l$ where the costs of a link l is c_l [1]. Other forms of the cost function are also possible [2].

It is easy to see that the optimal admission and routing decisions (1) are very sensitive to the uncertain route costs in a typical situation when utilities of several decisions are so close to each other that the best decision cannot be identified with a reasonable degree of confidence. This sensitivity may lead to instability of the minimum cost routing. Modeling this uncertainty by assuming that the link costs are random variables with some fixed probability distributions results in the admission and routing decisions that maximize the *average* utility [2]:

$$r_{opt} = \arg \max_{r \in \{\varnothing, R\}} E[u(c, r \mid w)] \tag{3}$$

However, even when the forms of the probability distributions for the resource costs can be reliably identified, the parameters of these distributions remain to be subject to uncertainty within the corresponding confidence regions, leaving the problems of sensitivity and instability unresolved.

III. Regularization of a Minimum Cost Routing

We propose to view randomization of the admission and routing decisions as a way to regularize solution to the ill-posed optimization problems (1) and (3). A game theoretic framework for regularization of optimization problem (1) has been proposed in [3]-[4]. This framework is based on the assumption that the route costs $c = (c_r : r \in R)$ are selected by an adversary within the given "confidence" region $c \in C$ in an attempt to maximize his utility function

$$L(c, r \mid w) = u(c, r_{opt} \mid w) - u(c, r \mid w) \tag{4}$$

This framework can be formalized as a two player, zero sum game of the network and the adversarial environment, where the network attempts to minimize losses (4) by selecting strategy $r \in \{\varnothing, R\}$. The optimal mixed network strategy can be interpreted as a regularized solution to the optimization problem (1). For a case of a single feasible route the corresponding game has been solved in [4].

We propose to generalize this game theoretic framework in the following two directions. First, it may be more realistic to assume that the cost of each link l is selected within some given interval $c_l \in [\breve{c}_l, \hat{c}_l]$ independently of the other link costs selections. Under this assumption the optimal admission and routing decisions are modeled as a non-cooperative game of $1 + L$ players, where L is the number of links in the network. Each link l attempts to maximize loss (4) by selecting $c_l \in [\breve{c}_l, \hat{c}_l]$. Second, if the cost of a link l is a random variables with some probability distributions which depends on some unknown parameter $q_l \in [\breve{q}_l, \hat{q}_l]$, utility (4) should be replaced with the average utility $\bar{L}(q, r \mid w) = E[L(c, r \mid w)]$ where vector $q = (q_l)$.

Currently, we are able to solve the corresponding games in a case of a parallel symmetric structure.

References

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[1] This work was supported by DARPA NMS program